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Discussion Papers

No. 18

August 2024

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ISSN 2194-2188

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Abstract

In this paper, we discuss how environmental damage and emission reduction policies affect the conduct of monetary policy in a two-sector (clean and dirty) dynamic stochastic general equilibrium model. In particular, we examine the optimal response of the interest rate to changes in sectoral inflation due to standard supply shocks, conditional on a given environmental policy. We then compare the performance of a nonstandard monetary rule with sectoral inflation targets to that of a standard Taylor rule. Our main results are as follows: first, the optimal monetary policy is affected by the existence of environmental policy (carbon taxation), as this introduces a distortion in the relative price level between the clean and dirty sectors. Second, compared with a standard Taylor rule targeting aggregate inflation, a monetary policy rule with asymmetric responses to sector-specific inflation allows for reduced volatility in the inflation gap, output gap, and emissions. Third, a nonstandard monetary policy rule allows for a higher level of welfare, so the two goals of welfare maximization and emission minimization can be aligned.

Keywords: climate change, environmental policy, inflation, macroeconomic stabilization, monetary policy

JEL classification: E32, E52, E58, Q54, Q58

1 Introduction

Since the Paris Agreement of 2015, climate policies have become central to the global economic agenda. In addition to governments and (supra)national public bodies, monetary authorities have also begun to express concern about the economic consequences of climate change, and they see the need for their proactive involvement. Speeches given in 2015 by Mark Carney, former governor of the Bank of England, and in 2019 by Mary C. Daly, president of the Federal Reserve Bank of San Francisco, are examples of this: both warned that climate change poses severe risks to economic development and financial stability, and could ultimately affect the conduct of monetary policy to achieve the goals of full employment and price stability. More recently (July 2021), the European Central Bank approved a new monetary policy strategy, claiming that central banks should commit to including the impact of climate change in their policy frameworks and be supportive of policy initiatives addressing environmental issues.

What effects do environmental policies and climate change have on the conduct of monetary policy? Many central banks around the world have recognized the implications that climate change may have for financial stability, envisioning macroprudential policy measures. However, little attention seems to be given to the implications for macroeconomic stabilization and monetary policy. In standard real business cycle (RBC) models, monetary policy plays no role. In RBC models with an environmental externality (Heutel 2012), which are often advanced to describe the impact of climate change on the economic system, an optimal environmental policy is designed to offset the negative externality. This usually takes the form of a carbon tax, as making emissions costly is seen as the most efficient way to reduce them (Nordhaus 1977). In an RBC model with an environmental externality and nominal frictions (Annicchiarico and Di Dio 2015), the instrument of monetary policy is added to compensate for the nominal frictions. However, an optimal carbon tax depends on business cycle shocks which cannot be identified in real time. What happens if the optimal tax is not feasible? In this paper, we focus on a scenario in which the world is constrained by a sub-optimal (exogenous) environmental policy affecting the business cycle, asking how this impacts the conduct of monetary policy. In particular we study how monetary policy should respond to standard shocks (e.g. cost push or TFP shocks) when carbon taxes and other environmental policies are in place. We do so by questioning the primary mandate of central banks' macroeconomic stabilization policy, asking whether stabilizing aggregate inflation is the optimal choice or, in the words of James (2021), we should distinguish "bad" inflation from "good" inflation. To answer this question, we analyze how the inclusion of environmental degradation and emissions reduction policies impacts the optimal values of monetary policy rule parameters in a two-sector macroeconomic model. We do this by studying how a nonstandard rule that distinguishes between "good" and "bad" inflation (here referred to inflation in a dirty sector and in a clean sector) performs compared to a conventional monetary rule in terms of welfare and emissions.

We develop a dynamic stochastic general equilibrium (DSGE) model augmented with a simplified climate module that features two intermediate sectors: a *clean* green sector and a *dirty* polluting sector. The purpose is to study how the distinction between a polluting and a non-polluting industry affects the efficient design of monetary policy when it is combined with different climate policies that address environmental externalities. In this framework, the monetary authority controls the nominal interest rate, which affects consumption, output and prices. At the same time, climate change and environmental policies (set by the government) can affect economic development (growth, relative prices and inflation volatility) and welfare. On the one hand, higher emissions result in more damage to productivity, which reduces labor and capital incomes and thus households' consumption possibilities. On the other hand, emission reduction policies (emission cap and carbon tax) introduce a distortion that ultimately leads to a reduction in consumption if it is not exactly offsetting the environmental friction. Taken together, these two factors can reduce the welfare level. Since the final objective of the central bank is to maximize social welfare, environmental issues cannot be deemed unrelated to monetary policy. In this work, we focus on the price stability target in a Taylor-rule-based policy (Taylor 1993, 1999) and the trade-off between inflation and output gap stabilization, and limiting polluting emissions. We look at sectoral rather than aggregate prices, assuming that the central bank can differentiate between clean and dirty good inflation when setting its policy. In the main simulation, we explore the response of the economy to exogenous sector-specific markup shocks. We show that optimal monetary rule parameters are affected by the type of environmental policy implemented and depend on which sector is hit by the shock.

The bridge in the literature between general equilibrium models and the environment is the integrated assessment model (IAM). The pioneering work of William Nordhaus (Nordhaus 1977, 2010; Nordhaus and Sztorc 2013), with his *Dynamic Integrated model of Climate and the Economy* (DICE), can be considered the forerunner of the wide strand of literature developed in recent decades around IAMs. DICE is an analytical model, designed as a policy optimization tool, that tries to represent the interconnection between the climate and the global economic system. In addition to the neoclassical economic growth theory, on which it is grounded, the "negative natural capital" of carbon concentrations is included (Nordhaus 2010). Environmental policies that aim to reduce anthropogenic emissions are therefore intended as investments to reduce this negative capital. A strand of literature combining environmental issues with the real business cycle theory is also based on the early work of Nordhaus. Heutel (2012) formally stresses the importance of business cycles in driving public policies: his idea is that, in order to design focused interventions to address climate change, it is important to build a model in which climate policies are explicitly integrated with macroeconomic fluctuations. He develops a DSGE model in which pollution appears as a stock variable that negatively affects the economy. This kind of integrated model has been defined as an environmental DSGE (E-DSGE) model.

The early E-DSGE literature focused primarily on the different effects of specific public environmental policies on the business cycle. Extensions of the model constructed by Heutel (2012) have been created by adding uncertainty (Hassler and Krusell 2018), New Keynesian nominal rigidities (Annicchiarico and Di Dio 2015; Annicchiarico and Dio 2017) and financial (Carattini et al. 2023) or labor-market frictions and migration (Chan 2019; Gibson and Heutel 2023). More recently, attention has shifted to the interaction between environmental and monetary policies and the possible extension of the tools available to central banks. In this respect, Chen et al. (2020) and Chan (2020) develop a climate-augmented monetary policy rule by adding an emission target to the standard Taylor equation. They find that such a monetary policy can create a conflict between welfare and climate objectives, as emission reductions can be recessive. Nakov and Thomas (2023) analyze the implication of what they define a climate-conscious (Ramsey optimal) monetary policy when the carbon tax is initially set at a sub-optimal level, showing that Central Banks's conventional interest rate policy is an inefficient instrument for reducing CO_2 emissions. Del Negro et al. (2023) show how climate policies can create a trade-off between inflation and output stabilization in presence of a shock to the carbon taxation, and how this result depends on the difference in price flexibility between a dirty and a green sector. Other scholars instead focus on the composition of central banks' balance sheets, developing models with financial frictions in which clean-quantitative easing programs are enforced (Diluiso et al. 2021; Ferrari and Nispi Landi 2023). We contribute to this literature by showing how monetary policy should be conducted when environmental policy and climate frictions affect the business cycle, in presence of standard shocks, and by asking whether monetary policy should actually be concerned with aggregate price stability or whether central banks' optimal responses to price changes in the clean and dirty sectors should be asymmetric.

Our main results are as follows: first, the optimal monetary policy is affected by the existence of environmental policy (carbon taxation), as this introduces a distortion in the relative price level between the clean and dirty sectors. Second, compared with a standard Taylor rule (TR) targeting aggregate inflation, a monetary policy rule with asymmetric responses to sector-specific inflation allows for reduced volatility in the inflation gap, output gap, and emissions. Third, a nonstandard TR allows for a higher level of welfare, so the two goals of welfare maximization and emission minimization can be aligned.

The paper is organized as follows: in section 2, we design our E-DSGE model by integrating a two-sector DSGE model with climate damage and introducing environmental fiscal policies. In section 3, we conduct an analysis of the impulse response functions (IRFs) in the E-DSGE model: for simplicity, we restrict our attention to a single environmental scenario (tax policy) and run the model under different monetary regimes[1]. In section 4, we derive the optimal monetary rule parameter values for our E-DSGE model with two sectors. In the first part of section 4 (subsection 4.1), we compare inflation, emissions and output gap volatility under the standard and the nonstandard monetary rules when exogenous shocks hit the economy simultaneously. In the second part of section 4 (subsection 4.2), we look at optimal monetary rule

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[1] For completeness, the results of simulations conducted under alternative environmental regimes are shown in the appendix.

coefficients, welfare loss and emission variation when shocks hit the economy asymmetrically. The last section concludes the paper.

2 Environmental-DSGE model

In this section, we describe our two-sector DSGE model augmented with environmental externalities. Building on the work of Annicchiarico and Di Dio (Annicchiarico and Di Dio 2015; Annicchiarico and Dio 2017) and Ferrari and Nispi Landi (2023), we integrate the Environmental-DSGE model of Heutel (2012) with New Keynesian nominal rigidities and an intermediate firm level. In this model, three types of firms operate in the economy: a final-consumption-good producer, which bundles "clean" and "dirty" outputs and operates in a perfectly competitive market; two intermediate firms aggregating, respectively, "clean" and "dirty" differentiated goods using a CES aggregator technology; and the "dirty" polluting firms[2] and "clean" non-polluting firms operating in a monopolistically competitive regime. The latter employ capital and labor as production factors. A sketch of the model is shown in Figure 1.

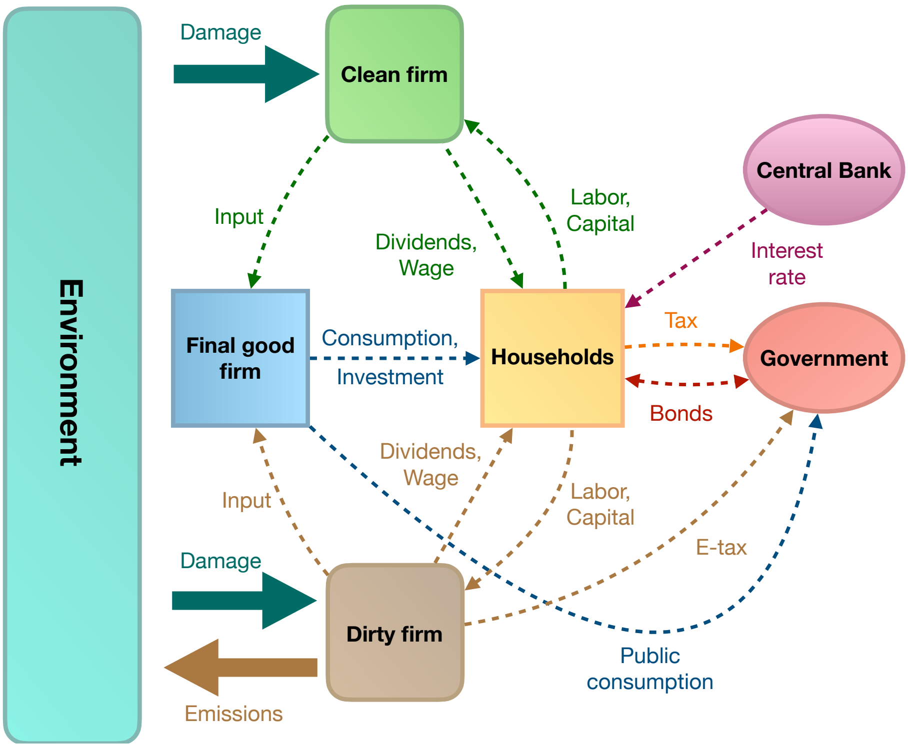

Figure 1: Scheme of the E-DSGE model.

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[2] Although in Heutel (2012) all firms pollute, in our model, only dirty firms pollute.

2.1 Households

Households are all identical and indexed by $i \in [0,1]$, they consume a consumption good and supply differentiated labor and capital. The utility of the representative infinitely lived household is:

$$U = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\varphi_c} - 1}{1-\varphi_c} - \psi \frac{l_t^{1+\varphi_l}}{1+\varphi_l} \right), \tag{1}$$

where l_t represents working hours and c_t is per-capita consumption (at time t); the period utility is characterized by a constant relative risk aversion (CRRA), where φ_c is the inverse elasticity of intertemporal substitution; ψ weighs the disutility of working; φ_l is the inverse of the Frisch elasticity; and β is the intertemporal discount factor. Households maximize the lifetime utility (equation (1)) subject to the budget constraint. For the budget of households, we differentiate capital, labor and wage by sector such that

$$b_t(i) + c_t(i) + i_t(i) = b_{t-1}(i)\frac{r_{t-1}}{\pi_t} + \sum_j \left(r_t^{k,j}(i) k_{t-1}^j(i) \right) + \sum_j \left(w_t^j(i) l_t^j(i) \right) - t_t + \mathcal{T}_t, \tag{2}$$

where $j = \{C, D\}$ stands for clean and dirty sectors. Here, b_t represents bond holdings, i_t represents investment, $b_{t-1}r_{t-1}$ denotes revenues from holding bonds, π_t is the consumer price index (CPI) inflation rate; $r_t^{k,j} k_{t-1}^j$ is the income from sector-specific capital service and $w_t^j l_t^j$ is the income from sector-specific labor; t_t is a lump-sum tax and $\mathcal{T}_t$ represents profits from firms ownership (equal to zero because of the perfect competition regime in which the firms operate). All variables are expressed in real terms[3]. Following the example of Christiano et al. (2005), we also consider the existence of implicit adjustment costs in investment[4], which makes adjusting the investment level in response to a departure of capital from its optimal level costly. By doing this, investment is smoothed over time. Households can also choose whether to invest in green or dirty sector capital. The law of motion of capital with quadratic

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[3] Lowercase letters denote real variables, uppercase letters nominal variables.

[4] In their work, Christiano et al. (2005) do not specify a functional form for the investment adjustment cost; instead, they specify some properties that the function should have. The functional form we choose is in line with those properties.

adjustment costs is

$$k_t^j(i) = (1-\delta)k_{t-1}^j(i) + i_t^j(i)\left[1 - \frac{\phi_i}{2}\left(\frac{i_t^j(i)}{i_{t-1}^j(i)} - 1\right)^2\right], \tag{3}$$

where $\phi_i > 0$ denotes the investment cost parameter. Investment and labor supply for the two sectors aggregate according to the following equations:

$$l_t(i) = \left[\left(l_t^D(i)\right)^{1+\varphi_h} + \left(l_t^C(i)\right)^{1+\varphi_h}\right]^{\frac{1}{1+\varphi_h}}, \tag{4}$$

$$i_t(i) = \left[i_t^D(i) + i_t^C(i)\right], \tag{5}$$

where $\varphi_h > 0$ represents the willingness of households to substitute labor between sectors. By setting this parameter higher than zero, we allow imperfect labor mobility across sectors (Cantelmo and Melina 2023). Individual labor supply varieties are aggregated using the Dixit-Stiglitz function (Dixit and Stiglitz 1977).

$$l_t^j = \left(\int_0^1 \left(l_t^j(i)\right)^{\frac{\xi_W - 1}{\xi_W}} di\right)^{\frac{\xi_W}{\xi_W - 1}}. \tag{6}$$

where ξ_W is the elasticity of substitution between individual labor supply. The latter is defined as a function of individual to aggregate wage share

$$l_t^j(i) = \left(\frac{w_t^j(i)}{w_t^j}\right)^{-\xi^W} l_t^j, \tag{7}$$

such that the aggregate intermediate wage reads

$$w_t^j = \left(\int_0^1 w_t(i)^{1-\xi_W} di\right)^{\frac{1}{1-\xi_W}}. \tag{8}$$

Additionally we introduce sectoral nominal wage rigidity in the form of quadratic adjustment costs à la Rotemberg (1983)[5]:

$$AC_{W,t}^j(i) = \frac{\phi_W}{2}\left(\frac{W_t^j(i)}{W_{t-1}^j(i)} - \bar{\pi}\right)^2 l_t^j W_t^j, \tag{9}$$

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[5] As an alternative, we could have employed the model of pricing of Calvo (1983), but we opted for Rotemberg's model because it is a more parsimonious model. Moreover, Rotemberg and Calvo models deliver equivalent dynamics when they are log-linearized around a zero-inflation steady state (Ascari and Rossi 2012).

which is added to the left-hand side of the budget constraint (2). Here ϕ_W represents the nominal wage adjustment cost parameter. Households' utility maximization problem yields the following first-order conditions (FOCs) with respect to $c_t(i)$, $w_t^j(i)$ and $k_t^j(i)$:

$$\begin{aligned}
c_t^{-\varphi_c} &= \lambda_t, && (10)\\
{mrs^j}_t &= \frac{\psi\, l_t^{\phi_l-\phi_h} {l^j}_t^{\phi_h}}{\lambda_t} && (11)\\
{\pi^j}_{W,t}\left({\pi^j}_{W,t}-\bar{\pi}\right) &= \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{{l^j}_{t+1}}{{l^j}_t}\frac{{w^j}_{t+1}}{{w^j}_t}{\pi^j}_{W,t+1}\left({\pi^j}_{W,t+1}-\bar{\pi}\right)\\
&\quad+\frac{\xi_W}{\phi_W}\left(\frac{{mrs^j}_t}{{w^j}_t}-\frac{\xi_W-1}{\xi_W}\right), && (12)\\
q_t^j &= \mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\beta\left((1-\delta)q_{t+1}^j+r_{t+1}^{k,j}\right)\right\}, && (13)
\end{aligned}$$

where:

$$\begin{aligned}
{\pi^j}_{W,t} &= \frac{W_t^j}{W_{t-1}^j}\\
&= \frac{W_t^j}{p_t}\frac{p_t}{p_{t-1}}\frac{p_{t-1}}{W_{t-1}^j} && (14)\\
&= \frac{w_t^j}{w_{t-1}^j}\pi_t.
\end{aligned}$$

and mrs_t^j is the inverse of the marginal rate of substitution between consumption and leisure; ${\pi^j}_{W,t}$ is the sectoral wage inflation; q_t measures the marginal value of capital with respect to consumption and is known as Tobin's q. From the FOC for b_t, we obtain the Euler equation:

$$\lambda_t = \beta\mathbb{E}_t\left(\lambda_{t+1}\frac{r_t}{\pi_{t+1}}\right), \tag{15}$$

where inflation π_{t+1} is defined as

$$\pi_{t+1} = \frac{p_{t+1}}{p_t}. \tag{16}$$

Lastly, the FOC with respect to i_t^j yields

$$\begin{aligned}
1 &= q_t^j\left[1-\frac{\phi_i}{2}\left(\frac{i_t^j}{i_{t-1}^j}-1\right)^2-\phi_i\left(\frac{i_t^j}{i_{t-1}^j}-1\right)\frac{i_t^j}{i_{t-1}^j}\right]\\
&\quad+\beta\mathbb{E}_t q_{t+1}^j\left[\frac{\lambda_{t+1}}{\lambda_t}\phi_i\left(\frac{i_{t+1}^j}{i_t^j}-1\right)\left(\frac{i_{t+1}^j}{i_t^j}\right)^2\right]. && (17)
\end{aligned}$$

From equation (15), we can write the real interest rate r_t^r as

$$r_t^r = \frac{r_t}{\pi_{t+1}}. \tag{18}$$

2.2 Firms

2.2.1 Final-good firms

Final good firms employ clean (y_t^C) and dirty (y_t^D) intermediate goods as inputs in a constant elasticity of substitution (CES) aggregator function:

$$y_t = \left[(1-\Delta)^{\frac{1}{\epsilon}} \left(y_t^C\right)^{\frac{\epsilon-1}{\epsilon}} + \Delta^{\frac{1}{\epsilon}} \left(y_t^D\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{19}$$

where y_t^C and y_t^D are goods from industry C and industry D, respectively, Δ is a weighting parameter and ϵ represents the elasticity of substitution between the two[6]. The final-good firm solves an intratemporal maximization problem to determine the optimal input combination:

$$\max_{y_t^C, y_t^D} \quad p_t y_t - \left[P_t^C y_t^C + P_t^D y_t^D\right],$$

where P_t^C and P_t^D are sector-specific prices and p_t represents the aggregate price index (CPI). This problem yields the following demand functions:

$$y_t^C = y_t (1-\Delta) \left(\frac{P_t^C}{p_t}\right)^{-\epsilon}, \tag{20}$$

$$y_t^D = y_t \Delta \left(\frac{P_t^D}{p_t}\right)^{-\epsilon}. \tag{21}$$

The aggregate price is

$$p_t = \left[(1-\Delta)\left(P_t^C\right)^{1-\epsilon} + \Delta \left(P_t^D\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{22}$$

We define $p_t^C \equiv \frac{P_t^C}{p_t}$ and $p_t^D \equiv \frac{P_t^D}{p_t}$ as the price of clean and dirty goods in terms of the aggregate CPI.

Both demand functions are increasing in terms of the final good production and price and decreasing in terms of their own price. Both demand functions depend on the composition of y, which is given by the exogenous value of Δ. For simplicity, we set $\Delta = 0.5$, such that the economy is split into two perfectly symmetric sectors. We define intermediate goods C and D as an aggregation of intermediate inputs $y_t^{C,D}(i)$

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[6] A unitary elasticity of substitution yields the classic Cobb-Douglas function; instead, we choose a value $\epsilon > 1$, which makes the two inputs imperfect substitutes.

produced by a continuum of monopolistically competitive firms indexed with i, with constant elasticity of substitution:

$$y_t^j = \left(\int_0^1 \left(y_t^j(i) \right)^{\frac{\xi_t^j - 1}{\xi_t^j}} di \right)^{\frac{\xi_t^j}{\xi_t^j - 1}}. \tag{23}$$

where ξ_t^j is the sector-specific elasticity of substitution parameter and y_t^j is the Dixit-Stiglitz aggregator. ξ_t^j is also a stochastic process that describes a markup shock to the inflation equation, as defined by Smets and Wouters (2003). It evolves following an AR(1) process:

$$\ln \xi_t^j = \rho_\xi \ln \xi_{t-1}^j + (1 - \rho_\xi) \ln \overline{\xi^j} - e_{\xi,t}^j, \tag{24}$$

with $0 < \rho_\xi < 1$ and $e_{\xi,t}^j \sim i.i.d.N\left(0, \sigma_{j,M}^2\right)$, where $\overline{\xi}$ is the steady-state level of the elasticity parameter and $e_{\xi,t}$ is an exogenous markup shock. The intermediate input demand is

$$y_t^j(i) = \Delta \left(\frac{P_t^j(i)}{P_t^j} \right)^{-\xi_t^j} y_t^j. \tag{25}$$

By substituting the values of $y_t^{C,D}$ from equations (20) and (21) into the intermediate input demand function (equation (25)), we obtain

$$y_t^C(i) = \left(\frac{P_t^C(i)}{P_t^C} \right)^{-\xi_t^C} y_t(1-\Delta) \left(\frac{P_t^C}{p_t} \right)^{-\epsilon}, \tag{26}$$

$$y_t^D(i) = \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\xi_t^D} y_t \Delta \left(\frac{P_t^D}{p_t} \right)^{-\epsilon}. \tag{27}$$

The zero-profit condition for intermediate-good firms then requires the aggregate intermediate price index to be

$$P_t^j = \left(\int_0^1 P_t(i)^{1-\xi_t^j} di \right)^{\frac{1}{1-\xi_t^j}}. \tag{28}$$

2.2.2 Clean and dirty intermediate-input firms

Clean and dirty firms employ sector-specific labor and capital in their production; additionally, the TFP in both industries is negatively affected by pollution. Dirty-good firms differ from clean-good producers in that they face a trade off between paying a tax on their negative externalities and using only a fraction of their production to abate their polluting emissions. The production function of the two sectors is

$$\begin{aligned} y_t^j(i) &= A_t^j \left(k_{t-1}^j(i) \right)^\alpha \left(l_t^j(i) \right)^{1-\alpha}, \qquad (29) \\ \text{and} \quad A_t^j &= (1 - D_t(x))\, a_t^j, \qquad (30) \end{aligned}$$

where A_t^j is the total factor productivity (TFP) net of the damage function D_t, and a_t^j is the industry-specific technology factor; α represents the input share parameters. Technology in both sectors evolves following an AR(1) process:

$$log(a_t^j) = (1-\rho_a)log(\bar{a}^j) + \rho_a log(a_{t-1}^j) + e_a^j, \tag{31}$$

where $\bar{a}^j$ is the steady-state level of technology and e_a^j is a sector-specific exogenous productivity shock[7].

The linkage between production and the climate is expressed via the damage function and abatement spending. Here, we define the functions that make up the climate module:

$$\begin{aligned} D_t(x_t) =& d_0 + d_1 x_t + d_2 x_t^2, && (32)\\ x_t =& \eta x_{t-1} + e_t + e_t^{ROW}, && (33)\\ e_t =& (1-\mu_t) h(y_t^D(i)), \quad \text{with} \quad \mu_t \in [0,1], && (34)\\ z_t =& g(\mu_t) y_t^D(i). && (35) \end{aligned}$$

The damage D_t is a quadratic function of the pollution stock x_t. The pollution stock is a function of domestic emissions e_t and emissions from the rest of the world e_t^{ROW}; η is a parameter describing the decay rate of atmospheric pollution. Domestic emissions are a function of polluting firms' production and abatement; μ_t is the fraction of emissions abated; and z_t represents the total abatement spending. In addition, we define two auxiliary functions,

$$\begin{aligned} h\left(y_t^D(i)\right) =& \gamma_1 \left(y_t^D(i)\right)^{1-\gamma_2}, \quad \text{with} \quad \gamma_2 < 0 < \gamma_1 \leq 1, && (36)\\ g(\mu_t) =& \theta_1 \mu_t^{\theta_2}, && (37) \end{aligned}$$

such that industrial emissions are an increasing and convex function of the output.

Nominal price rigidities are modeled again by introducing quadratic adjustment costs ($AC_t(i)$) à la Rotemberg (1983), which intermediate firms pay whenever they adjust their price with respect to the steady-state level of inflation:

$$AC_t^j(i) = \frac{\phi_p}{2} \left(\frac{P_t^j(i)}{P_{t-1}^j(i)} - \bar{\pi} \right)^2 y_t^j P_t^j. \tag{38}$$

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[7] This is the standard expression for TFP in DSGE models; however, in this paper we abstract from technology shocks.

In both sectors, firms pay the price adjustment cost and maximize their profits, expressed in terms of the CPI level p_t. The profit function of dirty-good producers is as follows:

$$\Pi_t^D(i) = \frac{P_t^D(i)}{p_t} y_t^D(i) - \tau_t^E(i) e_t(i) - z_t(i) - w_t^D L_t^D(i) - r_t^{k,D} k_{t-1}^D(i) \frac{AC_t^D(i)}{p_t}, \tag{39}$$

where $\Pi_t^D(i)$ is the dirty-firm profit and τ_t^E is the carbon tax on industrial emissions that polluting firms pay to the central fiscal authority. From the profit maximization problem for polluting firms, we get the following FOCs with respect to k_t^D, l_t^D and μ_t:

$$r_t^{k,D} = mc_t^D A_t^D \alpha \left(k_{t-1}^D\right)^{\alpha-1} (l_t^D)^{1-\alpha}, \tag{40}$$

$$w_t^D = mc_t^D A_t^D (1-\alpha) \left(k_{t-1}^D\right)^{\alpha} (l_t^D)^{-\alpha}, \tag{41}$$

$$\tau_t^E \gamma_1 \left(y_t^D\right)^{-\gamma_2} = \theta_1 \theta_2 \mu_t^{\theta_2 - 1}, \tag{42}$$

where mc_t^D is the sectoral Lagrangian multiplier related to marginal costs[8]. The dirty Phillips curve is

$$\begin{aligned} \pi_t^D \left(\pi_t^D - \bar{\pi}\right) = {} & \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{p_{t+1}^D y_{t+1}^D}{p_t^D y_t^D} \pi_{t+1}^D \left(\pi_{t+1}^D - \bar{\pi}\right) \right] \\ & + \frac{\xi_t^D}{\phi_p} \left[\frac{mc_t^D}{p_t^D} - \frac{\xi_t^D - 1}{\xi_t^D} + \tau_t^E (1-\mu_t) \gamma_1 (1-\gamma_2) \frac{\left(y_t^D\right)^{-\gamma_2}}{p_t^D} + \theta_1 \frac{\mu_t^{\theta_2}}{p_t^D} \right], \end{aligned} \tag{43}$$

where:

$$\begin{aligned} \pi_t^j &= \frac{P_t^j}{P_{t-1}^j} \\ &= \frac{P_t^j}{p_t} \frac{p_t}{p_{t-1}} \frac{p_{t-1}}{P_{t-1}^j} \\ &= \frac{p_t^j}{p_{t-1}^j} \pi_t. \end{aligned} \tag{44}$$

and p_t^D is the relative price of dirty good. Note that since dirty and clean firms incur different costs, they will not set the same price even in equilibrium; hence, we cannot simplify the price variables P_t^D and p_t. Marginal costs depend on the degree of elasticity between intermediate goods within the same sector.

For clean-good producers, the resource constraint is

$$\Pi_t^C(i) = \frac{P_t^C(i)}{p_t} y_t^C(i) - w_t^C L_t^C(i) - r_t^{k,C} k_{t-1}^C(i) - \frac{AC_t^C(i)}{p_t}. \tag{45}$$

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[8] Since in symmetric equilibrium all firms within the same sector choose the same price, we can drop the index i

As before, the profit maximization problem yields the following FOCs with respect to k_t^C and l_t^C:

$$r_t^{k,C} = mc_t^C A_t^C \alpha \left(k_{t-1}^C\right)^{\alpha-1} (l_t^C)^{1-\alpha}, \tag{46}$$

$$w_t^C = mc_t^C A_t^C (1-\alpha) \left(k_{t-1}^C\right)^{\alpha} (l_t^C)^{-\alpha}. \tag{47}$$

The clean Phillips curve is

$$\pi_t^C \left(\pi_t^C - \bar{\pi}\right) = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{p_{t+1}^C y_{t+1}^C}{p_t^C y_t^C} \pi_{t+1}^C \left(\pi_{t+1}^C - \bar{\pi}\right)\right] + \frac{\xi_t^C}{\phi_p}\left[\frac{mc_t^C}{p_t^G} - \frac{\xi_t^C - 1}{\xi_t^C}\right]. \tag{48}$$

2.3 Public sector and market clearing

Government consumption g_t is financed by both income t_t and emissions taxes. The public sector budget constraint with a zero net supply of bonds is

$$g_t = t_t + \tau_t^E e_t + b_t - (1 + r_{t-1})\, b_{t-1}, \tag{49}$$

$$\text{where} \quad t_t = \omega y_t. \tag{50}$$

t_t is defined as a fixed share of the income y_t, which is determined by the parameter ω. The good-market clearing condition implies that

$$y_t = c_t + g_t + i_t + z_t + \sum_j \left[\frac{\phi_p}{2}\left(\pi_t^j - \pi_t\right)^2 y_t^j p_t^j + \frac{\phi_W}{2}\left(\pi_{W,t}^j - \bar{\pi}\right)^2 l_t^j w_t^j\right]. \tag{51}$$

2.3.1 Standard monetary rule

The monetary authority follows a feedback rule of the Taylor rule class[9] to set the nominal interest rate in response to changes in the inflation rate. The standard TR is

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}}\right)^{\rho_m} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{y_t}{y^*}\right)^{\phi_y}\right]^{1-\rho_m}, \tag{52}$$

where $\bar{r}$ and $\bar{\pi}$ represent the corresponding Ramsey steady state of the nominal interest rate and inflation; y^* is the flexible price output[10]; ρ_m denotes the degree of monetary policy inertia. The ratio $\frac{y_t}{y^*}$ is the output gap and represents the deviation of output

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[9] The choice of a monetary rule stems from the fact that this type of rule-based policy is readily implementable and easy to communicate to the public. In contrast, a socially optimal policy designed by a Ramsey planner is not directly operational. Schmitt-Grohé and Uribe (2007) have shown that the outcomes of the two approaches, in terms of welfare maximization, can be extremely close.

[10] The output level obtained in the absence of of wage and price nominal rigidities.

from its flexible counterpart. ϕ_π and ϕ_y denote the response of the interest rate to inflation and output gap variation.

2.3.2 Nonstandard monetary rule

Given the structure of our model, we propose an alternative monetary policy rule, in which the inflation target is replaced by two targets, which correspond to the relative inflation of the two sectors. So, while maintaining the usual mandate of price stability, the central bank can set its policy rate taking into account the relative changes of P_t^C and P_t^D instead of the general level of the price p_t. By doing so, the coefficients of the monetary policy rule can be set independently. The nonstandard TR reads

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}}\right)^{\rho_m} \left[\left(\frac{\pi_t^C}{\bar{\pi}}\right)^{\phi_\pi^C} \left(\frac{\pi_t^D}{\bar{\pi}}\right)^{\phi_\pi^D} \left(\frac{y_t}{y^*}\right)^{\phi_y}\right]^{1-\rho_m}, \tag{53}$$

where ϕ_π^C and ϕ_π^D denote the response of the interest rate to the sector-specific inflation variation.

2.4 Environmental dynamics and policies

In RBC studies with an environmental production externality (e.g., Heutel (2012)), an optimal carbon tax is derived that can compensate for this externality. But what if an optimal carbon tax is not feasible? In this work, we define three environmental regimes characterized by different policies[11]:

1. *baseline*: the government does not impose any tax on polluting emissions, nor there is the incentive for firms to abate, such that $\tau_t^E = \mu_t = 0$;
2. *tax policy*: the government levies an emissions tax whose value is fixed such that $\tau_t^e = \bar{\tau}_e$ and abatement is constant[12];
3. *cap policy*: the government set a permanent limit on emissions based on a fixed amount of pollution stock, i.e. $e_t = \bar{x}(1-\eta) - e^{row}$; tax and abatement adjust

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[11] In the appendix we show results for an additional fourth environmental policy as in Annicchiarico and Di Dio (2015), i.e. a *target policy*. With this policy, an emission intensity target, i.e. the amount of emissions by unit of final output, is fixed by the public authority, such that $t^e = \frac{e_t}{y_t}$ and emission tax and abatement consistently adapt.

[12] In Heutel (2012) the carbon tax is procyclical.

accordingly. This policy is basically a simplified version of the cap-and-trade system implemented within the EU, in which the carbon budget is determined periodically by the relevant authorities and the price of carbon permits (the carbon tax) is determined by market demand.

In all three cases, the emission tax represents the major instrument used to offset the environmental friction. Note that $\bar{\tau}_e$ is predetermined by the government and does not correspond to an efficient Ramsey tax[13]. This means that it may not completely internalize the environmental externality, as it is not optimized to do so. In this work, we focus on an economy with sub-optimal climate policies, such that the intervention of monetary policy, besides being necessary to offset nominal frictions (as in a standard NK-RBC model), can contribute to offsetting the distortion caused by the environmental damage[14].

When no climate policy is set, we shut down the environmental tax and the abatement effort, such that $\tau_t^E = \mu_t = 0$. In doing this, the public sector is not endowed with any policy instruments to correct the distortion, and dirty firms do not internalize their negative externalities; they keep producing at the same pace regardless of the amount of CO_2 they emit. However, setting the emission tax equal to 0 and keeping the value of the weighting parameter Δ the same affects the steady-state level of the pollution stock, which increases significantly.

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[13] Many countries, including the U.S., set their carbon tax equal to the social cost of carbon (SCC), that is, the marginal cost to society (e.g., economic or human health costs) associated with the emission of an extra unit (tonne) of greenhouse gas. However, the carbon price is susceptible to political decisions, as the estimation of the SCC is dependent upon the public administration in charge. This explains why we have seen the SCC in the United States drop from $36 under the Obama administration to $7 during the Trump administration.

[14] In a keynote speech at the ILF conference on Clean Banking and Clean Central Banking, the President of the ECB, Christine Lagarde, stated that "Relying on just one solution, or on one party" (in our case, the environmental tax of the fiscal authority) "will not be enough to avoid a climate catastrophe" (Lagarde 2021). Therefore, monetary policy can also play an important role in combating climate change.

2.5 Calibration and steady state

The model is calibrated at a quarterly frequency in the U.S., mainly relying on the parametrization commonly used in the RBC and New Keynesian literature. The parameter ψ is calibrated such that the steady state labor supply is equal to 0.33 (1/3 of a day or 8 hours). Other specific ratios observed in the data are met, like the investment/output ratio (around 24%) and the government expenditure to GDP ratio (ca. 10%). Climate module parameters are updated according to the most recent available data, following Gibson and Heutel (2023) and Carattini et al. (2023). In their work, Gibson and Heutel (2023) employ the updated version of Nordhaus' DICE 2016R2 model to estimate the parameters of the damage, emissions and abatement cost functions. There is an extensive discussion in the literature about the magnitude of the damage. According to some experts, Nordhaus' damage function underestimates the impact of the pollution stock on the economy (Howard and Sterner 2017). In addition, temperature increases due to environmental degradation would affect different parts of the world asymmetrically and would even benefit some countries in the northern part of the globe (Kalkuhl and Wenz 2020). Nevertheless, investigating the nature of the damage function is beyond the scope of our work, so we follow the standard approach from the E-DSGE literature. The magnitude of the shocks is set to standard levels[15]. We set the standard error of the markup shock to -0.18, to induce a (negative) variation of the parameter ξ by one unit. The calibrated parameters are reported in Table 2. For comparison, we report in Table 1 the steady-state values of some variables of interest in E-DSGE models under the three environmental policy scenarios. We fix $\Delta = 0.5$, such that the intermediate goods sector is equally split between clean and dirty firms[16]. In this way, we want to isolate the transmission mechanism of monetary policy from the sector composition. The steady-state value of τ_t^E is numerically

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[15] One of the objectives of the paper is to show how the asymmetric monetary rule performs with respect to the standard TR, rather than finding the exact response parameters. Hence, although estimated shocks would fit reality better, they would not alter the nature of the results.

[16] According to the U.S. Environmental Protection Agency, the economic sectors that contribute the most to greenhouse gas (GHG) emissions in the United States are transportation, utilities, industry and manufacturing, commercial and residential, and agriculture. The sum of the GDP shares of these sectors for 2021 (data from the U.S. Bureau of Economic Analysis) amounts to 35%.

determined such that the initial amount of carbon stock is equal to ca. 820 $GtCO_2$ (similar to the level of global atmospheric carbon stock over the last decade, according to the update version of Nordhaus' DICE model) and $\Delta = 0.5$[17]. This value is common to all the environmental regimes considered here. For the baseline scenario only, $\bar{x}$ increases to ca. 1120, in order to keep the parameter Δ constant and equal to 0.5. The steady-state value of the emission tax is 0 with no environmental policy, and it is 0.012 otherwise. It is expressed in the arbitrary units of the model. In an E-DSGE model, the economy performs worse under the no-policy scenario: the reason for this is that the absence of an environmental policy leads to higher emissions and a larger pollution stock. When no environmental policy is implemented, production and prices of dirty and clean goods are the same in the two sectors, since they are perfectly identical and face the same marginal costs. On the contrary, in the model with a positive emission tax, dirty production is lower and the price of dirty goods is higher than that of clean goods. The comparison of welfare level reveals that, at the steady state, the introduction of a positive carbon tax increases welfare by 4.05%. This is due to the smaller pollution stock and, consequently, the lower environmental damage/GDP ratio.

3 Impulse responses in the E-DSGE model

We simulate our E-DSGE model to track the dynamic response of the endogenous variables to temporary negative sector-specific markup shocks[18]. In what follows, we restrict our attention to a single environmental policy (tax policy) and run the model under different monetary regimes. The same simulation is also conducted with alternative environmental regimes, and the results are displayed in Appendix E. The model is solved by computing the first-order Taylor approximation around the deterministic steady state of the linearized model, using the perturbation method. The model is

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[17] Here, we follow an approach similar to that taken by Ferrari and Nispi Landi (2023). In their work, the authors fix the initial level of carbon stock and the emission tax such that the value of Δ is 0.35. This value reflects the intermediate input composition in the EU, accounting for the current level of air pollution, which was calculated by Papoutsi et al. (2021).

[18] A negative shock implies that the elasticity of substitution ξ_t^j between intermediate inputs, within each sector, goes down, meaning the overall markup is higher and increases inflation.

Table 1: Steady-state variables for two-sector DSGE and E-DSGE models

			E-DSGE	
Variable	Description	Baseline	Tax Policy	Cap Policy
c/y	Consumption to GDP ratio	65.01%	64.71%	64.71%
i/y	Investment to GDP ratio	24.99%	24.83%	24.83%
g/y	Government exp. to GDP ratio	10.00%	10.40%	10.40%
D/y	Damage to GDP ratio	2.40%	0.98%	0.98%
e	Emissions	0.6539	0.4790	0.4790
p^D	Dirty good relative price	1.0	1.0033	1.0033
p^C	Clean good relative price	1.0	0.9967	0.9967
mc^D	Dirty marginal cost	0.8333	0.8252	0.8252
mc^G	Clean marginal cost	0.8333	0.8306	0.8306
τ_e	Environmental tax	0.0	0.012	0.012
x	Pollution stock	1120.91	821.06	821.06
W	Welfare	-206.07	-197.73	-197.73
$W\%$	Welfare variation wrt baseline	/	+4.05%	+4.05%

solved using the MATLAB pre-processor Dynare (Adjemian et al. 2024).

We distinguish four scenarios: in the first one, the central bank sets its monetary policy according to a standard TR with an aggregate inflation target. In the next two scenarios, the monetary rule employed is the one with sectoral inflation targets, as described in equation (53): these two scenarios differ in that in one regime, the central bank targets only clean inflation ($\phi_\pi^C = 1.5$, $\phi_\pi^D = 0$), while in the other, it targets only dirty inflation ($\phi_\pi^C = 0$, $\phi_\pi^D = 1.5$). For simplicity, we will call these rules "clean rule" and "dirty rule", respectively. In the last scenario, a Ramsey planner maximizes households' utility function by optimally choosing the monetary policy instrument value.[19].

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[19] Unlike the TR, with Ramsey optimal policy the social planner would maximize the household's utility by choosing the value of a specific policy instrument (in this case r_t), under the equilibrium conditions of the decentralized economy and contingent on the observation of the nature of the shock, but not (necessarily) on the specific variables showing up in the policy rule. It is convention in the literature, however, to use simple rules as they represent a closer approximation to the actual policy

Table 2: Model parameters

Parameter	Description	Value	Source
β	Discount factor	0.9975	RBC literature
φ_c	Inverse elasticity of intertemporal sub.	2	Carattini et al. (2023)
φ_l	Inverse Frisch elasticity	1	–
φ_h	Inverse elasticity sectoral labor	1	–
ξ_W	CES parameter of labor supply	10	Cantelmo and Melina (2023)
ϕ_W	Wage adjustment cost	107.196	calibrated (Ascari and Rossi 2012)
ψ	Disutility of work	7.5	calibrated
δ	Capital depreciation	0.025	RBC literature
ϕ_i	Investment adjustment cost	10	NK-DSGE literature
ξ	Elasticity of sub. intermediate goods	6	NK-DSGE literature
ϵ	Elasticity of sub. dirty vs clean goods	2	Carattini et al. (2023)
Δ	Weight of dirty good	0.5	
ϕ_p	Price adjustment cost	58.2524	calibrated (Ascari and Rossi 2012)
α	Share of capital in production	1/3	RBC literature
d_0	Damage function constant	-0.026	Carattini et al. (2023)
d_1	Damage function linear parameter	3.6613e-5	–
d_2	Damage function quadratic parameter	1.4812e-8	–
θ_1	Abatement cost function coefficient	0.0334	–
θ_2	Abatement cost function exponent	2.6	–
γ_1	Shifter of emissions function	1	Doda (2014)
$1-\gamma_2$	Emissions elasticity	1.2	–
η	Pollution decay rate	0.9965	Allen et al. (2018)
$\phi_\pi^{C,D}$	Mon. pol. param. clean/dirty inflation	1.5	NK-DSGE literature
ϕ_π	Mon. pol. param. inflation	1.5	–
ϕ_y	Mon. pol. param. output gap	0.5	–
ρ_a	Persistence of TFP/markup shock	0.95	–
ρ_ξ	Persistence of markup shock	0.95	–
ρ_g	Public spending persistence	0.8	–
ρ_m	Mon. pol. inertia	0.2	–
$\sigma_M^{C,D}$	SD of sector-specific markup shock	0.18	calibrated

We observe that on impact, the interest rate reaction to a shock, regardless of the affected sector, is generally aggressive when it's the Ramsey planner that chooses the policy rate. In case of a positive markup shock, this translates into a drop in the interest rate. The initial reaction is instead positive or null when a monetary rule is employed. For all scenarios, the interest rate gap is almost closed at the end of the simulation period, except when a nonstandard monetary rule is to be employed. Depending on the sector affected by the shock, we observe two opposite cases. Under a dirty shock, the dirty rule leads the interest rate to remain persistently above its steady state. The reverse situation occurs when the shock hits the clean sector. Consider now the dynamics of production and emissions, remembering that the latter is proportional to the dirty intermediate production. As expected, the Ramsey policy allows a smaller reduction in output than other monetary rules, being optimized for maximizing households' welfare. However, since welfare depends on the consumption, a relatively higher demand for consumption good also implies a higher level of emissions due to production. Thus, the optimal Ramsey monetary policy is not necessarily the best one from the standpoint of emission restraint. Based on the sector involved, a nonstandard TR turns out to outperform the Ramsey policy. The deepest drop in production is observed when the rule targets only the positive inflation of the sector hit by the supply shock (a clean rule whit a clean markup shock, a dirty rule with a dirty shock). This results in a permanently positive interest rate, which slows the overheating economy faster. Emissions are at their lowest level when this mechanism is in place.

Let's make an example: with a clean markup shock, the demand for clean inputs is reduced, due to the higher price, and demand for cheaper dirty goods increases, regardless of the monetary policy in place. Emissions rise as a consequence of higher production in the dirty sector. The increase in demand for dirty goods also leads to an increase in dirty inflation. Given the imperfect substitutability between the two goods, the supply shock to the clean sector translates into a demand shock for the dirty sector. Except for the case of Ramsey's optimal rate, the largest increase in emissions is observed when a dirty rule is adopted. With this policy, the central

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actions of central banks.

bank's reaction is more moderate, as dirty inflation is negative or close to zero. This initially leads to a weak interest rate (which then becomes negative after 10 periods), and consequently mitigates the contraction in total output. The result is a relatively higher level of emissions (and thus of the damage). When the shock hits the dirty sector, the difference in emissions between monetary regimes is less pronounced. This time, targeting only dirty inflation leads to a persistent increase in the interest rate, which reduces the level of production and the resulting GHG emissions. Overall, a clear dynamic of the role of monetary policy emerges in the presence of sectoral markup shocks. Targeting only the sector hit by a markup shock (leaving the response to falling prices in the other sector on the back burner), leads to a stronger contraction in output and emissions, but also a lower level of welfare. Since the optimal monetary policy (see the case of Ramsey's policy) aims at welfare maximization, a conflict may arise concerning emissions reduction. As we will see in the next section, the presence of a damage function plays a minor role in influencing the level of welfare over the business cycle, although the two objectives can be aligned. Imperfect substitutability among intermediate goods also plays an important role.

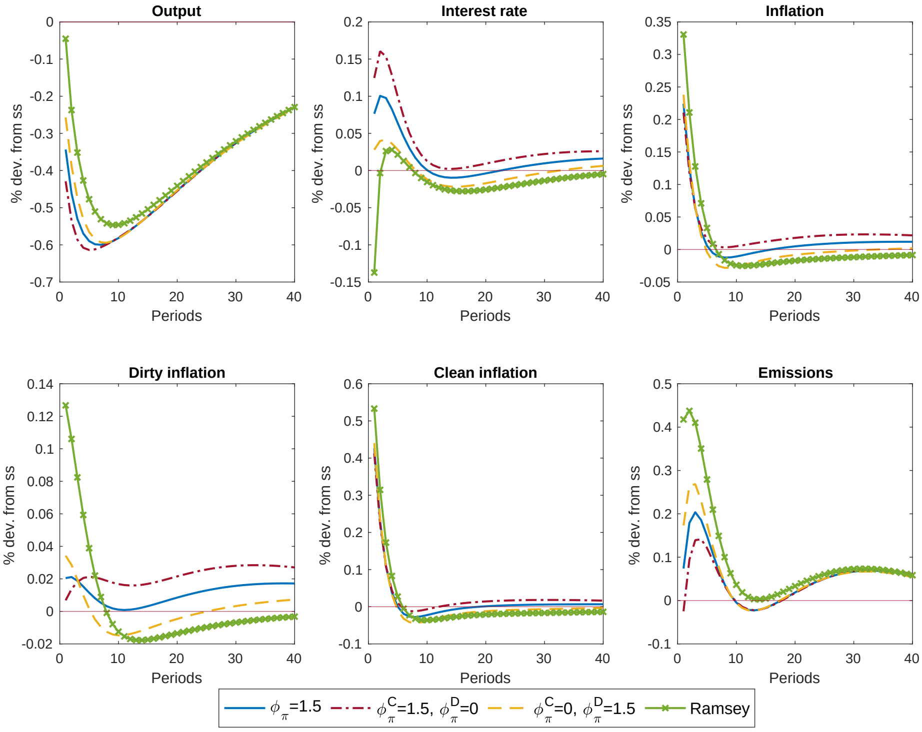

Figure 2: Clean markup shock—impulse responses under different monetary rules.

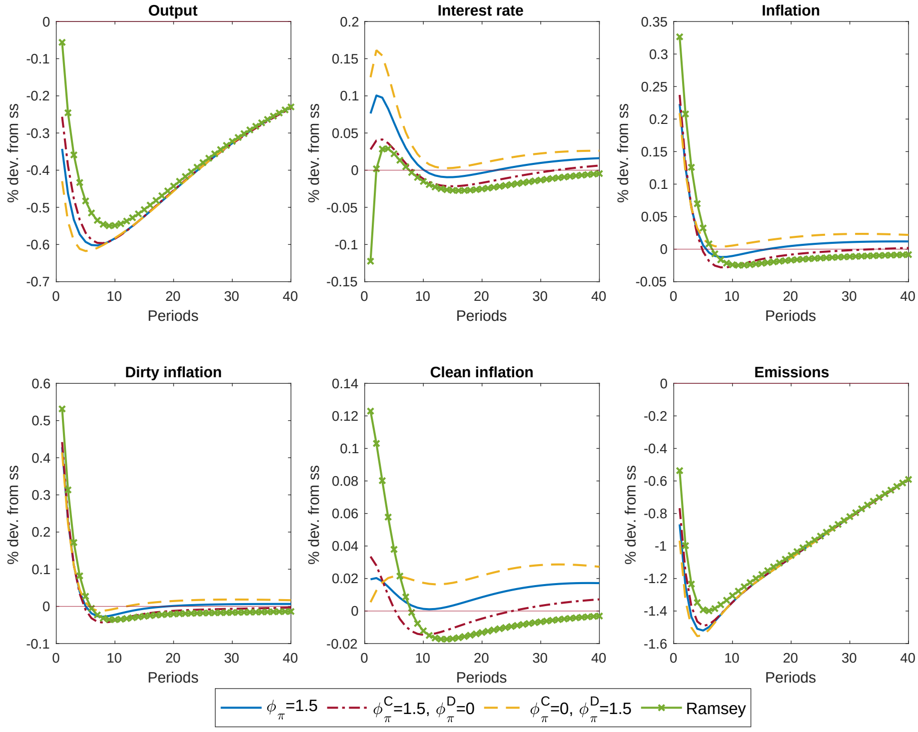

Figure 3: Dirty markup shock—impulse responses under different monetary rules.

4 Optimal monetary policy rule

Following the approach proposed by Schmitt-Grohé and Uribe (2007), we compute the optimal monetary rule (equations (52) and (53)) that maximizes the expected welfare of a representative household, conditional on it being in the steady state[20].

In the first part, we analyze the case in which a clean and a dirty (independent) markup shock occur simultaneously[21]. Later in this section, we will see what hap-

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[20] For the simulations, we derive a second-order Taylor approximation to the policy function of the model around its deterministic steady state using the perturbation method. The model is again solved with Dynare. For numerical optimization from here on, we employ a finite element grid with steps of 0.1, and 3 as the upper bound of the policy rule coefficients, as in Schmitt-Grohé and Uribe (2007).

[21] As emphasized earlier, the inflation target for both sectors is the same (which is also the aggregate inflation target). It is determined by the equilibrium conditions of the model, such that in the steady state prices do not vary (zero trend inflation). Our objective is to observe how the optimal coefficients of the monetary rule for each sector vary in response to fluctuations in the level of sectoral inflation. Setting a different inflation target for each sector would not change our experiment, since the coefficients would still be determined by the percentage change in the price level (which would

pens to the optimized parameters of the TR when the shock hits the clean and dirty intermediate sectors asymmetrically.

4.1 Symmetric shock

We start with the symmetric shock case. Both sectors are hit by uncorrelated cost push shocks (rising the price in both sectors) occurring simultaneously. We optimize the parameters of both standard and nonstandard TRs against welfare maximization, and repeat the same exercise for all the environmental policy scenarios. In figure 4 the trade-off between inflation and emissions volatility under the three policy regimes is presented. In each plot, the line represents pairs of standard deviation (SD) values of inflation and emissions, corresponding to an optimized standard TR. The small circles indicate all pairs of SDs corresponding to possible combinations of nonstandard TR parameters values. The area below the optimal line indicates that there are nonstandard TR parameter combinations for which both inflation and emission volatility are reduced. Comparison of environmental regimes also reveals that with a carbon tax policy the volatility of emissions is more limited (with a cap policy the volatility is zero because emissions do not vary by construction).

Similarly, when comparing inflation and output gap volatility (fig. 5), the nonstandard TR performs better than a standard one. In addition, we can see that with a tax policy regime (panel a) the volatility of prices and output is slightly higher than in the baseline case. The exact opposite is observed when a cap policy (panel b) is enforced. Thus, in general, a nonstandard TR can be superior in terms of inflation, emission and output gap volatility when maximizing welfare. Comparing environmental regimes, the cap policy turns out to be the best in reducing price and output fluctuations (central bank objectives), and the tax policy is second best in reducing emission fluctuations. The optimal coefficients of the two rules, conditional on the environmental regimes, are displayed in table 3.

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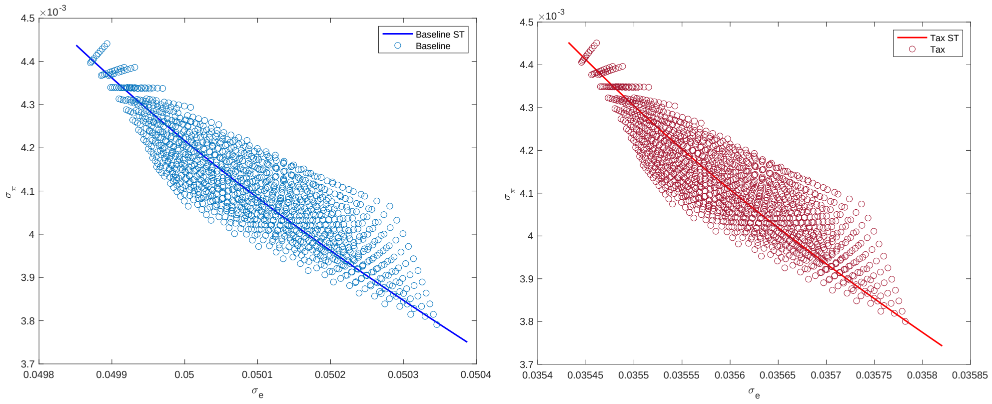
not change) and not by the starting point.

Table 3: Optimal monetary rule coefficients

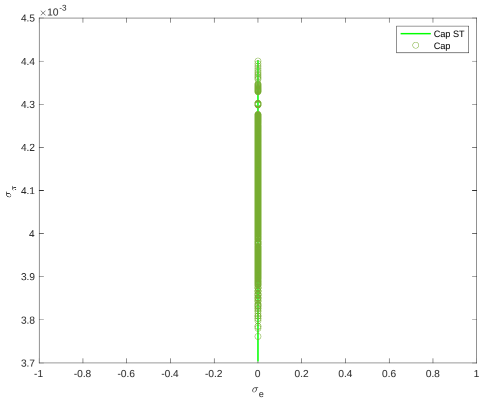
Env. Policy	Standard TR		Nonstandard TR		
	ϕ_π	ϕ_y	ϕ_π^C	ϕ_π^D	ϕ_y
Baseline	2.1	2.8	1.0	1.1	2.8
Tax policy	2.1	2.7	1.0	1.1	2.7
Cap policy	2.1	2.7	0.9	1.2	2.7

First column specifies the environmental regime. From second to sixth column, optimized parameter of the standard and non standard TR are displayed.

Figure 4: Inflation and Emissions Volatility



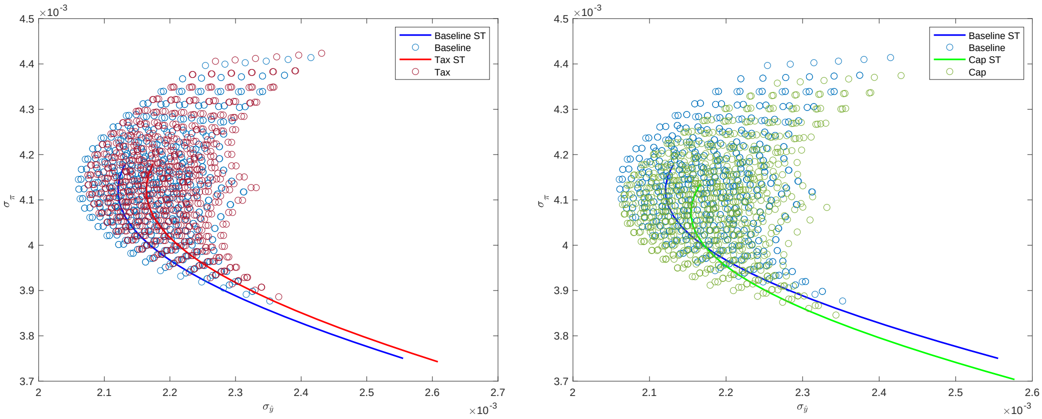
(a) Baseline policy (b) Tax policy



(c) Cap policy

Note: trade off between inflation and emissions volatility, standard (line) vs nonstandard (circle) TR; baseline (blue), tax (red) and cap (green) policy scenario with simultaneous markup shocks.

Figure 5: Inflation and Output Gap Volatility



(a) Baseline vs. tax policy

(b) Baseline vs. cap policy

Note: trade off between inflation and output gap volatility, standard (line) vs nonstandard (circle) TR; baseline (blue), tax (red) and cap (green) policy scenario with simultaneous markup shocks.

4.2 Asymmetric shocks

So far, we have analyzed the optimal response of monetary policy, through the lens of a monetary rule, when both (intermediate) economic sectors are buffeted by an inflationary markup shock. However, it is plausible to expect that not all sectors are affected in the same way or at the same time. What happens to the optimal monetary rule coefficients when the shock hits the two intermediate sectors asymmetrically?

The analysis is conducted as follows: first, a sectoral shock is simulated (i.e., affecting only one of the two sectors) and the optimal two-inflation-target TR coefficients are calculated. The procedure is repeated for each sector and each environmental policy scenario. Then, we perform the same procedure by replacing the previous monetary policy rule with a standard TR with a general inflation target. We compute households' welfare variation, which is expressed as a percentage of the steady-state consumption variation, or consumption equivalent (CE). Finally, we compare the levels of welfare and emission resulting from the implementation of the two optimized rules (see Appendix B for a full derivation). A positive (negative) value indicates that the implementation of a nonstandard TR results in higher (lower) welfare (emissions). The resulting optimized parameters, welfare and emission cost are displayed in Table

5. Here, we focus on the extreme cases in which the shock hits only one of the two intermediate sectors at a time.

Table 4: Optimal policy parameters: welfare maximization

Env. Policy	Shock	Standard TR		Nonstandard TR			W Δ%	E Δ%
		ϕ_π	ϕ_y	ϕ_π^C	ϕ_π^D	ϕ_y		
Baseline	C-Markup	2.1	2.8	0	2.2	0	0.0259	-2.1480
	D-Markup	2.1	2.8	2.2	0	0	0.0250	-2.1221
Tax policy	C-Markup	2.1	2.7	0	2.1	0	0.0266	-2.0634
	D-Markup	2.1	2.6	2.2	0	0	0.0264	-2.0544
Cap policy	C-Markup	2.1	2.9	0	2.1	0	0.0265	0
	D-Markup	2.1	2.6	2.2	0	0	0.0241	0

First column specifies the environmental regime. The letters C (clean) and D (dirty) in the shock column indicate in which sector the shock occurs. From third to seventh column, optimized parameter of the standard and non standard TR are displayed. Last two columns on the right represents welfare (W) and emissions (E) percentage variation (Δ%). The welfare (emissions) variation measures the benefit of implementing the two-sector-inflation-target rule vs a standard TR. Welfare (emissions) variation is expressed as a percentage of the steady-state consumption (emissions level) or consumption equivalent (CE). A positive value indicates that welfare (emissions level) is higher when a nonstandard TR is implemented.

When a standard TR is employed, the monetary authority reacts positively to both inflation and output gap. The value of the output gap coefficient slightly varies between environmental scenarios and (sectoral) shocks, while the inflation gap parameter remains unaltered. In the baseline scenario, coefficients are the same regardless of the sector affected by the shock. This suggests that the presence of the environmental externality (harm) alone is not sufficient to induce an asymmetric reaction of the monetary rule to sectoral inflation. Instead, whit a tax or a cap policy scenario, and so when a positive emission tax is introduced in the model, the reaction to sectoral shocks is asymmetric, though only slightly (the asymmetry is stronger with a cap policy). The presence of an emission tax in fact distorts the relative prices of the two goods (clean and dirty) and consequently the level of production, calling for a different reaction of monetary policy depending on the sector affected by the shock. When a nonstandard TR is implemented, the reaction to output gap is muted across all scenarios. The central bank reacts only to the inflation variation of the sector that is not hit by the shock: for example, when a clean markup shock occurs, only the dirty inflation parameter (ϕ_π^D) is positive and different from zero. The intuition is the following: the clean markup shock increases inflation and reduces output in the clean sector. Given the elasticity of substitution is higher than 1, final-good firms demand more dirty intermediate goods, and the dirty sector inflation goes up. This represents a supply shock for the clean sector, but a demand shock for the dirty sector: as monetary policy, in general, should react more to demand – rather than supply –driven inflation, the optimal coefficient on dirty inflation should be higher. The opposite is true in case of a dirty markup shock. In general we also see that the coefficient on clean inflation with a dirty shock is higher than the opposite case (stronger response to a demand shock in the clean sector). By being able to discriminate the sector in which the shock originates, the reaction to the output gap can be zeroed out, leading at the same time to an increase in the level of welfare.

Let us proceed with the evaluation of the results by considering the fluctuations in emissions. The nonstandard TR is shown to be consistently superior in terms of welfare maximization; the same can also be said with respect to emission minimization. Indeed, it is observed that the level of emissions in an economy with an unconventional rule is lower than it is in the case in which the central bank adopts a standard rule. The cap policy scenario is by construction an exception, as emissions do not vary under this environmental regime. The reduction in emissions when a nonstandard TR is implemented, ranges between 2.05 and 2.15%. Despite the existing trade-off between inflation and emissions volatility, the welfare maximization and emission minimization goals seems to be aligned. The fact that the reduction occurs even in the absence of emission taxation indicates that the damage caused by emissions from dirty production is enough to align the two objectives. The asymmetric reaction of monetary policy seems to allow in most of the cases for a downsizing of production in the dirty sector such that it leads to a relative reduction in emissions without being at the expense of welfare.

It is also clear from the table that, whatever environmental regime is in place, a two-sector TR is superior to a standard monetary rule, from a welfare perspective. The welfare gain of employing a nonstandard TR ranges between 0.024 and 0.027%

in terms of the consumption equivalent. It is worth noting that what really matters here is the sign, since the amount of welfare gain/cost depends strongly on the size of the shock. We can interpret this result in the following way: giving the monetary authority the ability to discriminate between clean and dirty inflation provides an opportunity to exploit the full potential of monetary policy in maximizing welfare. Different sectors face different costs, and the existence of a second-best environmental instrument (such as the emissions tax employed in our economy) that affects only one sector requires the support of another policy to alleviate the price distortion caused by that instrument and the environmental externality.

5 Conclusions

The relevance of climate change for economic stability has prompted all actors, including central banks, to review their policies concerning growth and business cycle stabilization. In this paper, we have investigated how environmental degradation and public policies aimed at counteracting this phenomenon influence the conduct of monetary policy. In particular, we have examined how they change the response of the interest rate (set by the central bank through a rule-based monetary policy) to sectoral price variations. To do this, we developed a two-sector New Keynesian DSGE model, augmented with a climate module and environmental friction, in the tradition of Heutel (2012). We observed that differentiating sectors with respect to their production costs and introducing environmental policies into the model causes the monetary authority to respond asymmetrically to changes in sectoral inflation, depending on the sector affected by the shock and on the specific environmental regime. The monetary rule with two inflation targets developed here proves to be superior to a standard TR and closer to an optimal Ramsey policy. The two intermediate goods are not perfect substitutes (a supply shock in one sector partially spreads to the other in the form of a demand shock), and a higher welfare level can require higher production and hence emissions. Nevertheless, the welfare maximization and emission minimization goals prove to be aligned. When a fixed cap is enforced, the unconventional TR brings higher welfare (compared to a standard rule) without affecting the environment with higher emissions. The cap policy turns out to be the best in reducing price and output fluctuations (which are the central bank objectives). A fixed carbon tax regime reduces the volatility of emissions, compared to the baseline scenario without environmental policies; but it also results in relatively higher variability of inflation and output gap.

Not all countries have introduced serious environmental policies to deal with climate change. However, a cap policy has already been widely implemented in Europe and in some states in the U.S. A monetary rule with an asymmetric response to sectoral inflation would be ideal here, as it would lead to welfare improvements.

As stated in a recent speech by the economic historian Harold James, "Not all price increases are the same, and some are desirable (...)" (James 2021). This somewhat corroborates the idea of a central bank supporting the ecological transition of the economy by accommodating (more) the variation of "certain" prices, while impeding the variation of "others". The feasibility of such an asymmetric reaction of the central bank depends on its ability to identify the cause associated with price variation and to distinguish its sector of origin. The set of information needed for a central bank to apply such a rule and address the climate change problem probably remains beyond the monetary authority's possibilities. Although hardly feasible, the fact remains that such a nonstandard monetary rule would provide additional support to at least stabilize emissions within the business cycle. This responds to the need for central banks to include the impact of climate change in the setting of monetary policy and to revise such policies with the intention of contributing to the green transition.

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Appendix

A E-DSGE Model Derivations

A.1 Households

The Lagrangian function for the household utility maximization problem is

$$\begin{aligned}\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t & \left\{ \left(\frac{c_t^{1-\varphi_c}-1}{1-\varphi_c} - \psi \frac{\left[\left(\left(l_t^D\right)^{1+\varphi_h} + \left(l_t^C\right)^{1+\varphi_h}\right)^{\frac{1}{1+\varphi_h}}\right]^{1+\varphi_t}}{1+\varphi_t} \right) \right. \\ & + \lambda_t \left[b_{t-1}\frac{r_{t-1}}{\pi_t} + k_{t-1} r_t^k + (w_t^C l_t^C + w_t^D l_t^D) - t_t + \mathcal{T}_t - b_t - c_t - i_t \right] + \\ & \left. + q_t \left[(1-\delta)k_{t-1} + i_t \left[1 - \frac{\phi_i}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2\right] - k_t\right]\right\},\end{aligned} \tag{54}$$

A.2 Intermediate Firms: Dirty and Clean Phillips curves

From the Lagrangian function of a dirty sector

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left\{ \frac{P_t^D(i)}{p_t} y_t \Delta \left(\frac{P_t^D}{p_t}\right)^{-\epsilon} \left(\frac{P_t^D(i)}{P_t^D}\right)^{-\xi_t^D} + \right. \\ & - \tau_t^E(i)\left(1-\mu_t(i)\right)\gamma_1 \left[y_t \Delta \left(\frac{P_t^D}{p_t}\right)^{-\epsilon} \left(\frac{P_t^D(i)}{P_t^D}\right)^{-\xi_t^D}\right]^{1-\gamma_2} + \\ & - \theta_1 \mu_t(i)^{\theta_2} \left[y_t \Delta \left(\frac{P_t^D}{p_t}\right)^{-\epsilon} \left(\frac{P_t^D(i)}{P_t^D}\right)^{-\xi_t^D}\right] + \\ & - w_t^D l_t^D(i) - r_t^{k,D} k_{t-1}^D(i) - \frac{\phi_p}{2}\left(\frac{P_t^D(i)}{P_{t-1}^D(i)} - \bar{\pi}\right)^2 \frac{y_t^D(i) P_t^D(i)}{p_t} + \\ & \left. + mc_t^D(i) \left[A_t^D(i) \left(k_{t-1}^D(i)\right)^{\alpha} \left(l_t^D(i)\right)^{1-\alpha} - y_t \Delta \left(\frac{P_t^D}{p_t}\right)^{-\epsilon} \left(\frac{P_t^D(i)}{P_t^D}\right)^{-\xi_t^D}\right]\right\},\end{aligned}$$

we derive the dirty Phillips curve by taking the FOC with respect to $P_t^D(i)$:

$$\begin{aligned}\frac{\partial L}{\partial P_t^D(i)} : \lambda_t & \left\{ \frac{y_t^D}{p_t}(1-\xi_t^D)\left(\frac{P_t^D(i)}{P_t^D}\right)^{-\xi_t^D} - \frac{\phi_p}{P_{t-1}^D(i)}\left[\frac{P_t^D(i)}{P_{t-1}^D(i)} - \bar{\pi}\right]\frac{y_t^D P_t^D}{p_t} + \right. \\ & + \xi_t^D mc_t^D(i) \frac{y_t^D}{P_t^D}\left(\frac{P_t^D(i)}{P_t^D}\right)^{-\xi_t^D-1} - \theta_1 \mu_t(i)^{\theta_2}\frac{y_t^D}{P_t^D}(-\xi_t^D)\left(\frac{P_t^D(i)}{P_t^D}\right)^{-\xi_t^D-1} + \\ & \left. - \tau_t^E(i)(1-\mu_t(i))\gamma_1 \frac{(y_t^D)^{1-\gamma_2}}{P_t^D}\xi_t^D(\gamma_2-1)\left[\frac{P_t^D(i)}{P_t^D}\right]^{\xi_t^D(\gamma_2-1)-1}\right\} + \\ & + \lambda_{t+1}\beta \mathbb{E}_t \left\{ \phi_p \frac{y_{t+1}^D P_{t+1}^D}{p_{t+1}}\left[\frac{P_{t+1}^D(i)}{P_t^D(i)} - \bar{\pi}\right]\frac{P_{t+1}^D(i)}{P_t^D(i)^2}\right\} = 0.\end{aligned}$$

At this point, we can drop the index i, since we are in a symmetric equilibrium, and simplify the equation:

$$
\begin{aligned}
(1-\xi_t^D) - \frac{\phi_p}{P_{t-1}^D}\left(\pi_t^D-\bar{\pi}\right)P_t^D + \xi_t^D mc_t^D\frac{p_t}{P_t^D} + \xi_t^D\tau_t^E(1-\gamma_2)(1-\mu_t)\gamma_1\frac{(t_t^D)^{-\gamma_2}p_t}{P_t^D} + \\
+\theta_1\mu_t^{\theta_2}\xi_t^D\frac{p_t}{P_t^D} + \frac{\lambda_{t+1}}{\lambda_t}\beta\mathbb{E}_t\left[\phi_p\frac{y_{t+1}^D P_{t+1}^D p_t}{p_{t+1}y_t^D}\left(\pi_{t+1}^D-\bar{\pi}\right)\frac{P_{t+1}^D}{{P_t^D}^2}\right] = 0
\end{aligned}
$$

By rearranging the terms and employing the definition of relative inflation as in equation 44, we obtain the dirty Phillips curve:

$$
\begin{aligned}
\pi_t^D\left(\pi_t^D-\bar{\pi}\right) = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{p_{t+1}^D y_{t+1}^D}{p_t^D y_t^D}\pi_{t+1}^D\left(\pi_{t+1}^D-\bar{\pi}\right)\right] + \\
+\frac{\xi_t^D}{\phi_p}\left[\frac{mc_t^D}{p_t^D} - \frac{\xi_t^D-1}{\xi_t^D} + \tau_t^E(1-\mu_t)\gamma_1(1-\gamma_2)\frac{\left(y_t^D\right)^{-\gamma_2}}{p_t^D} + \theta_1\frac{\mu_t^{\theta_2}}{p_t^D}\right].
\end{aligned}
$$

We follow the same procedure to derive the clean Phillips curve. We write the clean Lagrangian function as

$$
\begin{aligned}
\mathcal{L} =& \mathbb{E}_t\sum_{t=0}^{\infty}\beta^t\lambda_t\left\{\frac{P_t^C(i)}{p_t}y_t(1-\Delta)\left(\frac{P_t^C}{p_t}\right)^{-\epsilon}\left(\frac{P_t^C(i)}{P_t^C}\right)^{-\xi_t^C} + \right. \\
&-w_t^C l_t^C(i) - r_t^{k,C}k_{t-1}^C(i) - \frac{\phi_p}{2}\left(\frac{P_t^C(i)}{P_{t-1}^C(i)}-\bar{\pi}\right)^2\frac{y_t^C(i)P_t^C(i)}{p_t} + \\
&\left.+mc_t^C(i)\left[A_t^C(i)\left(k_{t-1}^C(i)\right)^{\alpha}\left(l_t^C(i)\right)^{1-\alpha} - y_t(1-\Delta)\left(\frac{P_t^C}{p_t}\right)^{-\epsilon}\left(\frac{P_t^C(i)}{P_t^C}\right)^{-\xi_t^C}\right]\right\}.
\end{aligned}
$$

Next, we take the FOC with respect to $P_t^C(i)$:

$$
\begin{aligned}
\frac{\partial L}{\partial P_t^C(i)} : \lambda_t & \left\{\frac{y_t^C}{p_t}(1-\xi_t^C)\left(\frac{P_t^C(i)}{P_t^C}\right)^{-\xi_t^C} - \frac{\phi_p}{P_{t-1}^C(i)}\left[\frac{P_t^C(i)}{P_{t-1}^C(i)}-\bar{\pi}\right]\frac{y_t^C P_t^C}{p_t} + \right. \\
&\left. +\,\xi_t^C mc_t^C(i)\frac{y_t^C}{P_t^C}\left(\frac{P_t^C(i)}{P_t^C}\right)^{-\xi_t^C-1}\right\} + \\
&+\lambda_{t+1}\beta\mathbb{E}_t\left\{\phi_p\frac{y_{t+1}^C P_{t+1}^C}{p_{t+1}}\left[\frac{P_{t+1}^C(i)}{P_t^C(i)}-\bar{\pi}\right]\frac{P_{t+1}^C(i)}{{P_t^C(i)}^2}\right\} = 0.
\end{aligned}
$$

Then, we drop the index i:

$$
(1-\xi_t^C) - \frac{\phi_p}{P_{t-1}^C}\left(\pi_t^C-\bar{\pi}\right)P_t^C + \xi_t^C mc_t^C\frac{p_t}{P_t^C} + \frac{\lambda_{t+1}}{\lambda_t}\beta\mathbb{E}_t\left[\phi_p\frac{y_{t+1}^C P_{t+1}^C p_t}{p_{t+1}y_t^C}\left(\pi_{t+1}^C-\bar{\pi}\right)\frac{P_{t+1}^C}{{P_t^C}^2}\right] = 0
$$

After rearranging the equation, we obtain the clean Phillips curve as follows:

$$
\pi_t^C\left(\pi_t^C-\bar{\pi}\right) = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{p_{t+1}^C y_{t+1}^C}{p_t^C y_t^C}\pi_{t+1}^C\left(\pi_{t+1}^C-\bar{\pi}\right)\right] + \frac{\xi_t^C}{\phi_p}\left[\frac{mc_t^C}{p_t^G} - \frac{\xi_t^C-1}{\xi_t^C}\right].
$$

B Optimal monetary policy: consumption equivalent variation

The optimal monetary policy is computed by maximizing households' utility function, and the welfare variation is measured in terms of the consumption equivalent as follows:

$$W_t^o = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U((1-\Upsilon)c_t, l_t), \tag{55}$$

$$W_t^o - W_t^b = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[U((1-\Upsilon)c_t, l_t) - U(c_t, l_t) \right],$$

$$W_t^o - W_t^b = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{((1-\Upsilon)c_t)^{1-\varphi_c} - 1}{1-\varphi_c} - \psi \frac{l_t^{1+\varphi_l}}{1+\varphi_l} - \frac{c_t^{1-\varphi_c} - 1}{1-\varphi_c} + \psi \frac{l_t^{1+\varphi_l}}{1+\varphi_l} \right],$$

$$\Upsilon = 1 - \left[\left(W_t^o - W_t^b \right) (1-\varphi_c)(1-\beta)(c)^{\varphi_c - 1} + 1 \right]^{\frac{1}{1-\varphi_c}}. \tag{56}$$

Here, Υ stands for the welfare cost of implementing a specific policy rule—denoted as optimal (o)—vs the baseline (b) policy, in terms of the CE.

C E-DSGE steady state

Output

$$y = \bar{y} \tag{57}$$

Capital stock

$$x = \bar{x} \tag{58}$$

Dirty relative price

$$p^D = \overline{p^D} \tag{59}$$

Technology

$$a^{C,D} = \bar{a} \tag{60}$$

Cost-push/elasticity of substitution

$$\xi^{C,D} = \bar{\xi} \tag{61}$$

Carbon tax

$$\tau^E = \overline{\tau^E} \tag{62}$$

Income tax

$$t = \omega \bar{y} \tag{63}$$

Public consumption

$$g = \tau^E + t \tag{64}$$

Inflation

$$\pi = \pi^{C,D} = \bar{\pi} \tag{65}$$

Nominal interest rate

$$r = \frac{\bar{\pi}}{\beta} \tag{66}$$

Real interest rate

$$r^r = \frac{1}{\beta} \tag{67}$$

Tobin's q

$$q^C = q^D = 1 \tag{68}$$

Rental rate of capital

$$r^k = \frac{1}{\beta} - (1-\delta) \tag{69}$$

Aggregate price (numeraire)

$$p = 1 \tag{70}$$

Emissions

$$e = \frac{x(1-\eta)}{6} \tag{71}$$

Abatement cost

$$\mu = \left[\frac{\tau^E \gamma_1 \left(\Delta y \left(p^D\right)^{-\epsilon}\right)^{-\gamma_2}}{\theta_1 \theta_2}\right]^{\frac{1}{\theta_2 - 1}} \tag{72}$$

Dirty production

$$y^D = \left[\frac{e}{\gamma_1(1-\mu)}\right]^{\frac{1}{1-\gamma_2}} \tag{73}$$

Damage function

$$D = \left[d_2(x)^2 + d_1(x) + d_0\right] \tag{74}$$

Clean TFP

$$A^C = (1-D)a^C \tag{75}$$

Dirty TFP

$$A^D = (1-D)a^D \tag{76}$$

Clean relative price

$$p^C = \left[\frac{1-\Delta(p^D)^{1-\epsilon}}{1-\Delta}\right]^{\frac{1}{1-\epsilon}} \tag{77}$$

Dirty demand

$$y^D = y\Delta \left(\frac{{P^D}^{-\epsilon}}{p} \right) \tag{78}$$

Clean demand

$$y^C = y(1-\Delta) \left(\frac{{P^C}^{-\epsilon}}{p} \right) \tag{79}$$

Dirty marginal cost

$$mc^D = \frac{\xi^D - 1}{\xi^D} p^D - \left[\theta_1 \mu^{\theta_2} + \gamma_1 \tau^E (y^D)^{-\gamma_2} (1-\gamma_2)(1-\mu) \right] \tag{80}$$

Clean marginal cost

$$mc^C = \frac{\xi^C - 1}{\xi^C} p^C \tag{81}$$

Dirty capital demand

$$k^D = \frac{\alpha y^D}{r_k^D} mc^D \tag{82}$$

Clean capital demand

$$k^C = \frac{\alpha y^C}{r_k^C} mc^C \tag{83}$$

Dirty labor demand

$$l^D = \left(\frac{y^D}{A^D (k^D)^\alpha} \right)^{\frac{1}{1-\alpha}} \tag{84}$$

Clean labor demand

$$l^C = \left(\frac{y^C}{A^C (k^C)^\alpha} \right)^{\frac{1}{1-\alpha}} \tag{85}$$

Dirty wage

$$w^D = \frac{(1-\alpha) y^D}{l^D} mc^D \tag{86}$$

Clean wage

$$w^C = \frac{(1-\alpha) y^C}{l^C} mc^C \tag{87}$$

Aggregate capital

$$k = k^C + k^D \tag{88}$$

Aggregate labor

$$l = \left[(l^C)^{(1+\phi_h)} + (l^D)^{(1+\phi_h)} \right]^{\frac{1}{1+\phi_h}} \tag{89}$$

Investment C

$$i^C = \delta k^C \tag{90}$$

Investment D

$$i^D = \delta k^D \tag{91}$$

Abatement spending

$$z = y^D \theta_1 \mu^{\theta_2} \tag{92}$$

Households' consumption

$$c = y - i - \bar{g} - z \tag{93}$$

Dirty labor marginal rate of substitution

$$mrs^D = \frac{\xi_W - 1}{\xi_W} w^D \tag{94}$$

Clean labor marginal rate of substitution

$$mrs^C = \frac{\xi_W - 1}{\xi_W} w^C \tag{95}$$

Dirty labor supply

$$mrs^d = \frac{\psi l^{\phi_l - \phi_h} l_d^{\phi_h}}{\lambda} \tag{96}$$

Clean labor supply

$$mrs^c = \frac{\psi l^{\phi_l - \phi_h} l_c^{\phi_h}}{\lambda} \tag{97}$$

D Sensitivity analysis

Let's consider how the optimized parameters of the two TR vary when changing some key parameters of our model. First, we set $\epsilon = 10$. A higher elasticity of substitution between clean and dirty goods in the production function implies a greater ability of the production system to absorb sectoral supply shocks, able to replace inputs more easily. The greater resilience thus makes it possible for the central bank to react more strongly to changes in the output gap (as reflected in the higher ϕ_y parameter value, regardless of the environmental policy in place). With a non standard TR, we also observe a weaker response to clean inflation, perfectly offset by a stronger one to dirty inflation. When changing the discount factor, the reaction of monetary policy becomes weaker. We test our model with a relatively low value of $\beta = 0.90$. As can be assumed, reducing the discount factor decreases the utility associated with future consumption (saving) by households. Monetary policy has a reduced effect on agents' welfare. The response to clean and dirty inflation, as well as to output gap variation is substantially weaker. The reaction of the central bank does not change substantially if we instead adjust the exponent of the emission function $\gamma_2 = 0.4$[22]. No significant differences are observed among the 3 environmental regimes.

Table 5: Sensitivity analysis of optimal monetary rule coefficients

Env. Policy	Parameters value	Standard TR		Nonstandard TR		
		ϕ_π	ϕ_y	ϕ_π^C	ϕ_π^D	ϕ_y
Baseline	Basic specification	2.1	2.8	1.0	1.1	2.8
	$\epsilon = 10$	2.1	3.0	0.6	1.5	3.0
	$\beta = 0.90$	1.1	0.1	0.6	0.6	0.4
	$\gamma_2 = 0.4$	2.1	2.9	1.0	1.1	2.9
Tax policy	Basic specification	2.1	2.7	1.0	1.1	2.7
	$\epsilon = 10$	2.1	3.0	0.8	1.3	3.0
	$\beta = 0.90$	1.1	0.1	0.6	0.6	0.4
	$\gamma_2 = 0.4$	2.1	2.7	1.0	1.1	2.7
Cap policy	Basic specification	2.1	2.7	0.9	1.2	2.7
	$\epsilon = 10$	2.1	3.0	0.6	1.5	3.0
	$\beta = 0.90$	1.1	0.1	0.6	0.6	0.4
	$\gamma_2 = 0.4$	2.1	2.7	1.0	1.1	2.7

First column specifies the environmental regime. Second column specifies alternative values of a specific model parameter. "Basic specification" parameters value are $\beta = 0.9975$, $\epsilon = 2$, $\gamma_2 = -2$. From third to seventh column, optimized parameter of the standard and non standard TR are displayed.

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[22] With this value, the emission function becomes concave, while in the main simulation it is convex.

E Additional IRF plots on comparison of monetary policies: tax, target, cap and baseline regimes

Tax policy

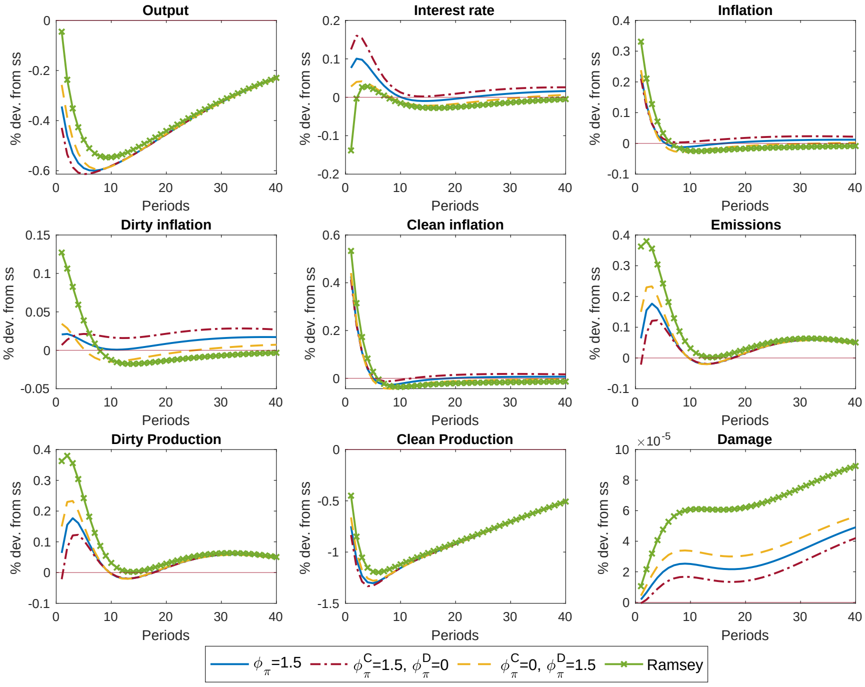

Figure 6: Clean markup shock—impulse response function under different monetary rules.

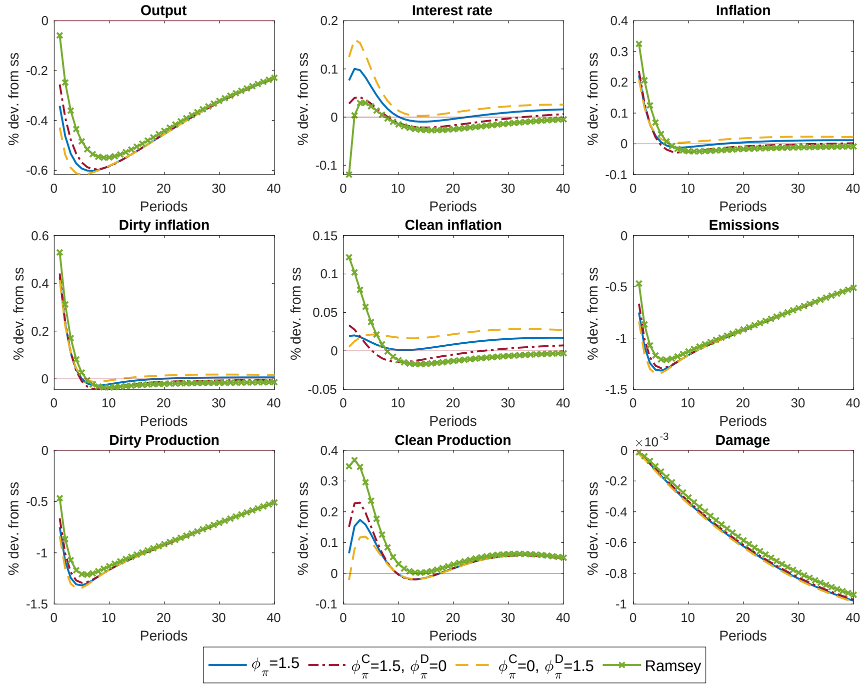

Figure 7: Dirty markup shock—impulse response function under different monetary rules.

Target policy

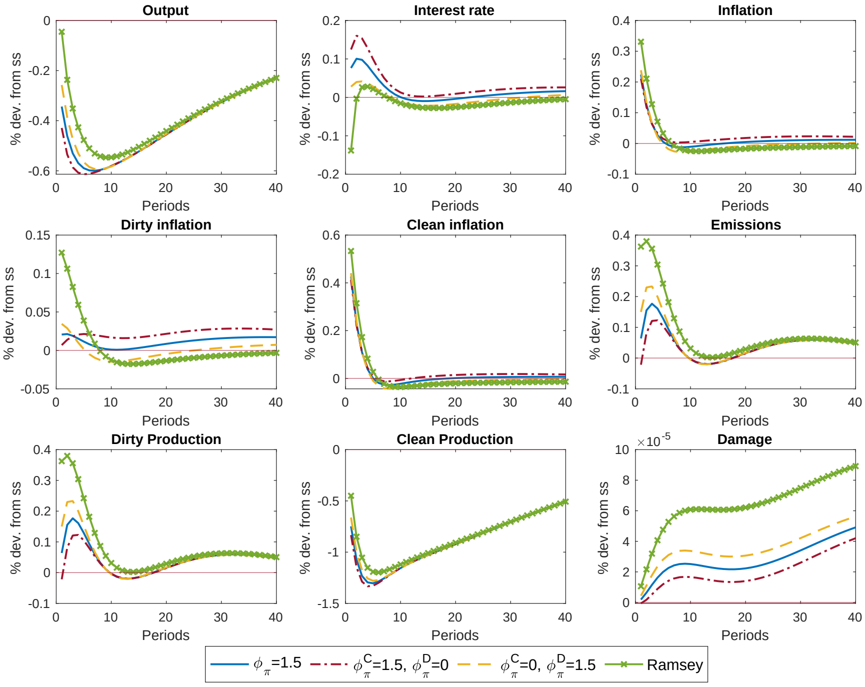

Figure 8: Clean markup shock—impulse response function under different monetary rules.

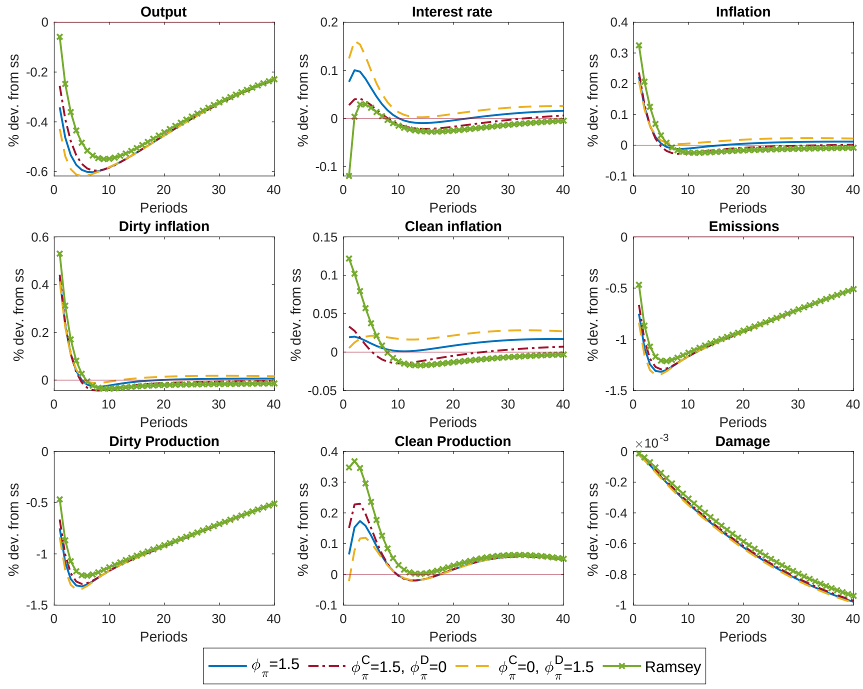

Figure 9: Dirty markup shock—impulse response function under different monetary rules.

Cap policy

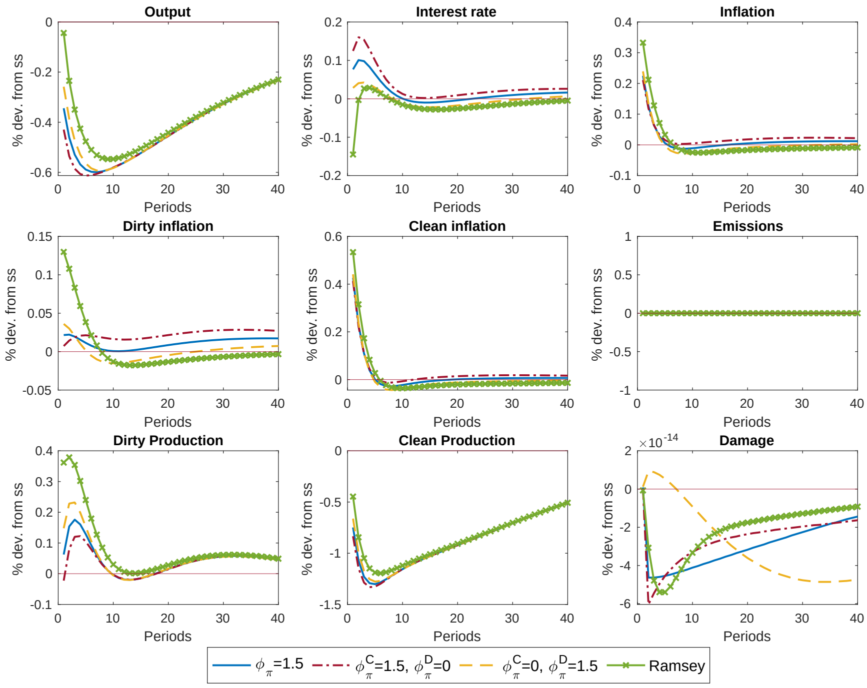

Figure 10: Clean markup shock—impulse response function under different monetary rules.

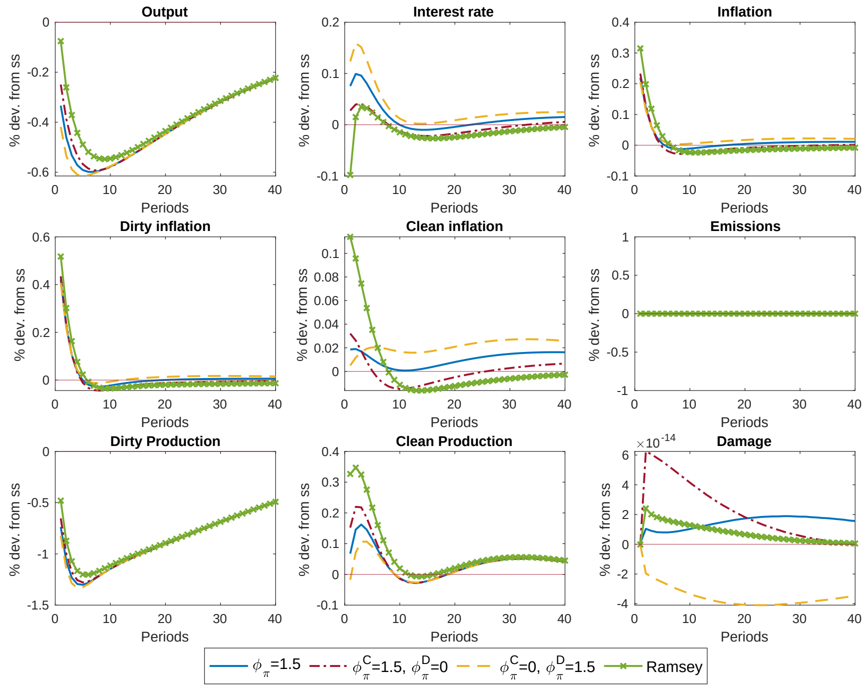

Figure 11: Dirty markup shock—impulse response function under different monetary rules.

Baseline

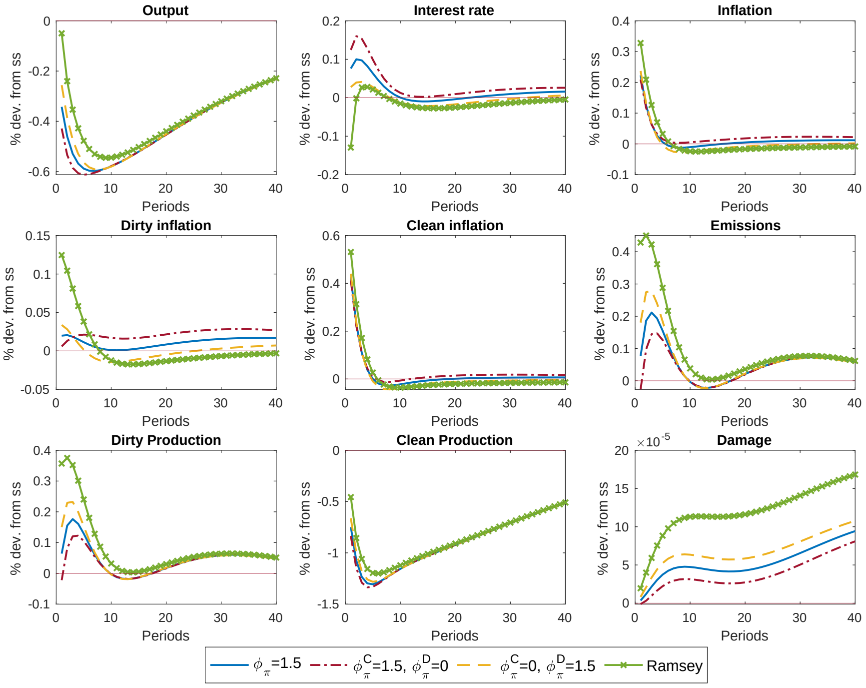

Figure 12: Clean markup shock—impulse response function under different monetary rules.

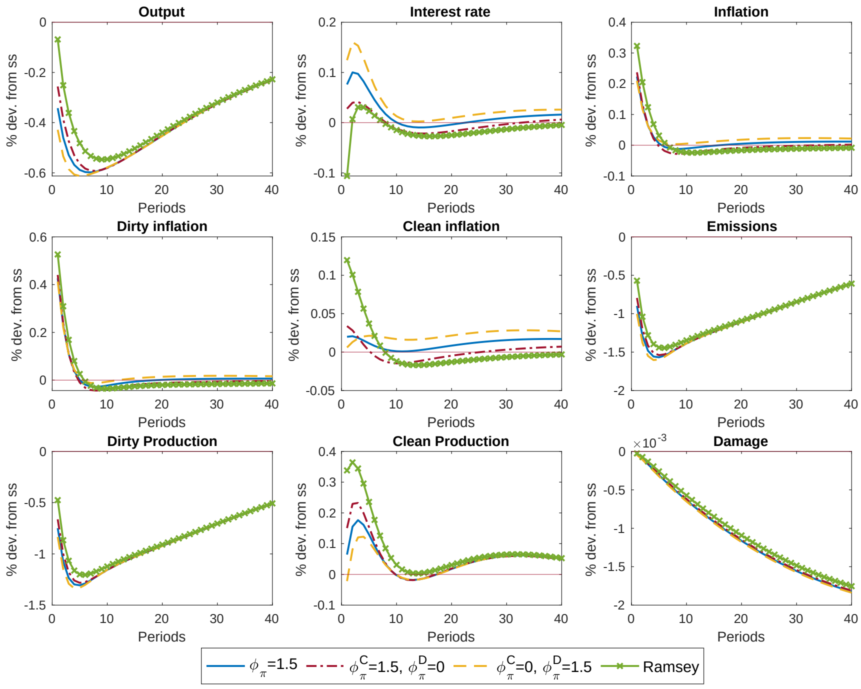

Figure 13: Dirty markup shock—impulse response function under different monetary rules.

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ISSN 2194-2188

Leibniz
Association

The IWH is funded by the federal government and the German federal states.